H7110 Project 2 Part 1

266720

Date: Friday 5^{th} November 2024

Abstract

This paper presents the design and analysis of a two-stage gearbox transmission, with a Continuously Variable Transmission (CVT) and a fixed gear stage, intended to deliver 5833 W with an input speed of 1000 RPM. Key design parameters include optimising torque, speed, and efficiency while addressing load, pressure, and material durability for both CVT and fixed stages. The study evaluates bearing types and gear configurations. Radial ball bearings are used for the CVT stage and cylindrical roller bearings for the fixed gear stage. Power requirements, force distributions, and Hertzian contact pressures help with material selection. Potential failure modes are analysed.

Nomenclature

δ	Maximum deflection of spring (mm)
η_{CVT}	Efficiency of CVT stage
η_{gear}	Efficiency of fixed-gear transmission
η_{total}	Overall efficiency of transmission system
λ_{eff}	Effective slenderness ratio
ω_{in}	Input angular velocity (rad/s)
ω_{out}	Angular velocity of output shaft (rad/s)
τ_y	Yield shear strength (Pa)
τ_{max}	Maximum shear stress (Pa)
a	Contact radius in Hertzian pressure calculation (mm)
a_{ISO}	Life adjustment factor for lubrication and contamination
C	Dynamic load rating (N)
C	Spring index ($C = \frac{D}{d}$)
D	Mean coil diameter of the spring (mm)
d	Diameter (mm)
d_p	Pitch diameter of a gear (mm)
D_{cyl}	Cylindrical housing diameter (mm)
E'	Effective modulus of elasticity for Hertzian contact (Pa)
E^*	Effective modulus of elasticity (Pa)
f	Face width of gear teeth (mm)
F_n	Normal force (N)
f_n	Natural frequency of the spring (Hz)
F_t	Tangential force (N)
F_{ball}	Force on each ball (N)
G	Shear modulus of elasticity (Pa)
I	Second moment of area of shaft cross-section (m^4)

i_{fixed} Fixed gear stage ratio

$i_{\max}$ Maximum CVT Ratio

$i_{\min}$ Minimum CVT Ratio

i_{total} Total effective transmission ratio

J Polar moment of inertia (m^4)

K_w Wahl correction factor

L Free height of spring (mm)

L_s Solid height of spring (mm)

L_{10} Expected bearing life (millions of revolutions)

M Bending moment (Nm)

N_{in} Input Speed (RPM)

ndm Bearing speed parameter (n is speed in RPM, d is bearing bore diameter in mm)

P Equivalent dynamic load (N)

P Output Power (W)

p Exponent for bearing type ($p = 3$ for ball bearings, $p = 10/3$ for roller bearings)

$p_{\max}$ Maximum Hertzian contact pressure (GPa)

R^* Effective radius of curvature (m)

R_{C1} Radius of bearing cage (mm)

T_{final} Final output torque (Nm)

T_{in} Input Torque (Nm)

T_{out} Output Torque (Nm)

W Maximum load on spring (N)

Contents

1. Declaration

This report is submitted as part requirement for the degree of Mechanical Engineering at the University of Sussex. It is the product of my own labour except where indicated in the text. The report may be freely copied and distributed provided the source is acknowledged. I hereby give permission for a copy of this report to be loaned out to students in future years.

2. Introduction

This project is the second part designing a two stage gear box, with a fixed stage and a CVT stage.

2.1. Objectives

(i) Select suitable gears for this transmission
(ii) Select suitable bearings for the fixed stage of the gearbox such that they have a minimum operational life of 20,000 hours
(iii) Design the input shaft of the gearbox (fixed stage); this should include the appropriate steps of shaft design as shown in the lecture notes or other sources. Specify the type of torque transfer element needed (no calculations are required).
(iv) The cost must be kept as low as possible

2.2. Project Description

2.2.1. Parameters

All the parameters from the first part have been carried over:
(i) Output power is 5833W
(ii) Input speed is 1000 RPM
(iii) i_{max} CVT is 0.750
(iv) i_{min} CVT is 0.525
(v) Ratio fixed gear stage is 0.698

3. Gears

3.1. Type of Gears

For this design, helical gears are chosen for being quieter and smoother power transmission. Helical gears. They are slightly less efficient due to sliding friction and generate axial forces, these can be managed with proper bearing selection.

3.2. Number of Teeth

The required gear ratio is 0.698. A MATLAB script (in the Appendix) shows suitable gear pairs within the specified constraints of 10 to 200 teeth. The script generates:

- **Pair 1:** Driver = 149, Driven = 10,
- **Pair 2:** Driver = 10, Driven = 104.

Verifying the combined ratio:

$$\frac{10}{149} \times \frac{104}{10} = 0.6980,$$

which matches the target ratio. The intermediate gear can be removed to improve efficiency. It simplifies the system and maintains performance.

3.3. Helical Gear Geometry

The helical gear design is based on the following parameters:

- **Module:** $m_t = 2\,\text{mm}$ (transverse plane),
- **Helix angle:** $\psi = 20°$,
- **Normal module:** $m_n = m_t \cos\psi = 1.88\,\text{mm}$,
- **Pressure angle:** $\alpha_n = 20°$.

The pitch diameters are calculated as:

$$d_p = z \cdot m_t,$$

where z is the number of teeth. For the 149-tooth driver:

$$d_p^{\text{driver}} = 149 \cdot 2 = 298\,\text{mm},$$

and for the 104-tooth driven gear:

$$d_p^{\text{driven}} = 104 \cdot 2 = 208\,\text{mm}.$$

The face width is designed to ensure sufficient overlap:

$$f = 1.5\,p_a = 1.5 \cdot \frac{m_t}{\tan\psi} = 15.6\,\text{mm},$$

which meets the condition $f \geq 1.5\,p_a$ for adequate contact.

3.4. Material Choice

The gears are carburized alloy steel, **AISI 8620**, for its high surface hardness, core toughness and ease of manufacture. The material is treated to achieve the following.

- Surface hardness: $58 - 62\,\text{HRC}$ for wear resistance,
- Core toughness to resist bending fatigue.

Alternative materials considered include nitrided steel (AISI 4340) and cast iron. However, these were rejected because of cost or insufficient toughness under operating conditions.

3.5. Stress Analysis

The gear design is checked for bending and contact stresses using the Lewis and AGMA equations. For bending stress:

$$\sigma_b = \frac{W_t}{f m_t J},$$

where:

- W_t: Tangential force, calculated as $W_t = \frac{2T}{d_p}$,
- J: Lewis form factor, selected based on equivalent teeth,
- f: Face width.

For contact stress:

$$\sigma_H = C_p \sqrt{\frac{W_n}{f d_p}},$$

where C_p is the material constant, and W_n is the normal force. The stresses are compared with the material limits for factor of safety.

3.6. Manufacturing and Heat Treatment

The manufacturing process includes:

1. **Gear Cutting:** Precision hobbing makes accurate tooth profiles,
2. **Heat Treatment:** Carburising for wear resistance,
3. **Post-Processing:** Grinding for a smooth surface finish, reducing noise and improving efficiency.

3.7. Dynamic Factor Adjustment

Given the moderate operating speed (1000 RPM), dynamic factors (K_v) are minimised with tight tolerances and surface finishes. The expected dynamic factor is:

$$K_v = 1.22,$$

suitable for high-precision ground gears.

3.8. Summary of Gear Design

- **Gear Type:** Helical,
- **Material:** AISI 8620,
- **Module:** $m_t = 2\,\text{mm}$,
- **Helix Angle:** $\psi = 20°$,
- **Face Width:** 15.6 mm,
- **Manufacturing Process:** Hobbing, carburising, and grinding.

4. Suitable Bearings

4.1. Rolling Element Bearings

Rolling element bearings are critical components in this design, providing support to rotating elements while minimising friction. They are designed to handle radial, axial, or combined loads with high precision and efficiency. Rolling bearings are selected based on load capacity, speed, alignment tolerance, and space constraints, as summarised in Table 1.

4.2. Bearing Design Considerations

Load and Speed: Bearings are selected to handle expected loads and speeds without exceeding their dynamic or static load ratings. Radial loads are predominant in both the CVT and fixed gear stage, with axial loads considered negligible. High-speed capabilities are necessary for the CVT.

Lubrication: Proper lubrication is critical for minimising friction and wear. For this design:

- Radial ball bearings in the CVT will use grease lubrication to simplify maintenance,
- Cylindrical roller bearings in the fixed gear stage will use an oil bath for durability under higher loads.

Fatigue Life: Bearing life is using the L10 rating; 90% reliability under specified conditions. Adjustments for contamination, misalignment, and lubrication quality are factored into life calculations.

4.3. CVT Stage (Discs and Balls)

The CVT stage requires bearings that handle:

- High speeds ($ndm > 1\,\text{million}$) for smooth operation,
- Moderate radial loads with minimal friction.

Radial Ball Bearings: Selected for their high-speed capability ($> 30,000\,\text{RPM}$), low friction, and efficiency (99%). These bearings are commonly used in applications requiring continuous rotation with minimal power loss.

Life Calculation: The dynamic load rating (C) for a bearing is determined using the ISO 281 standard:

$$L_{10} = \left(\frac{C}{P}\right)^p \cdot a_{\text{ISO}},$$

where:

- L_{10}: Expected life (in millions of revolutions),
- C: Dynamic load rating,
- P: Equivalent dynamic load,
- $p = 3$ (for ball bearings),
- a_{ISO}: Life adjustment factor for lubrication and contamination.

For this design, $L_{10} = 20\,\text{million rev}$, ensuring long service life.

4.4. Fixed Two-Step Gear Stage

This stage has higher load demands and moderate speeds.

Cylindrical Roller Bearings: These are chosen for their:

- High radial load capacity,
- Durability under heavy loads and moderate speeds ($ndm < 0.5\,\text{million}$),
- Alignment tolerance for fixed stages.

Material Selection: Bearings are made from through-hardened high-carbon chromium steel (AISI 52100). This material provides:

- High surface hardness ($58 - 63\,\text{HRC}$),
- Excellent fatigue resistance under dynamic loading,
- High wear resistance.

4.5. Manufacturing and Assembly

Bearings are precision-manufactured with tolerances down to $\pm 0.025\,\mu\text{m}$. Key processes include:

- Grinding and honing for smooth raceway finishes,
- Lapping of rolling elements to achieve sphericity and surface smoothness ($< 0.01\,\mu\text{m Ra}$),
- Final assembly with automated inspection for quality assurance.

Bearing Type	Load Capacity	Misalignment Tolerance	Speed Suitability	Application Examples
Ball	Moderate radial and light axial	Low	High	Motors, fans, pumps
Cylindrical Roller	High radial, no axial	Low	High	Gearboxes, heavy-duty conveyors
Spherical Roller	Heavy radial and some axial	High	Moderate	Mining, material handling
Tapered Roller	High radial and axial	Low	Moderate	Automotive, gear drives
Needle Roller	High radial, compact size	Low	Moderate	Transmissions, universal joints
Thrust	High axial only	Low	Low to moderate	Screw jacks, crane hooks

Table 1: Summary of Bearing Types and Their Applications

4.6. Summary of Bearing Design

- CVT stage: Radial ball bearings for high speed, low friction,
- Fixed gear stage: Cylindrical roller bearings for high radial loads,
- Lubrication: Grease for CVT bearings, oil bath for gear stage,
- Materials: AISI 52100 steel for durability and fatigue resistance,
- Service life: Bearings designed for $L_{10} \geq 20\,\text{million rev}$.

This combination ensures reliability and efficiency under the diverse operational conditions of the gearbox.

5. Shaft Design

5.1. Shaft Material Selection

The shaft material must withstand combined bending, torsional, and radial stresses while maintaining manufacturability and cost-efficiency. Based on the torque and load requirements, **medium carbon steel, AISI 1045**, is selected. This material offers:

- **Yield Strength:** 310 MPa, suitable for fatigue loads under cyclic bending.
- **Toughness:** Ensures resistance to stress concentrations and impact.
- **Machinability:** Facilitates efficient turning, milling, and threading, which are necessary for keyways and shoulder features.
- **Heat Treatment:** Induction hardening will be applied to the surface to achieve a hardness of 58 HRC, enhancing wear resistance and fatigue life.
- **Ultimate Strength:** $S_u = 565\,\text{MPa}$.

To enhance the fatigue life and wear resistance, the shaft will be surface-hardened through induction heating to a depth of 2 mm after machining.

5.2. Design Steps and Critical Points

The shaft must support a gear load of $W_2 = 2394\,\text{N}$ at $L_{\text{gear}} = 0.6\,\text{m}$ from the left bearing and transfer a steady torque of $T = 69.64\,\text{Nm}$. Bearings at the ends provide support at $x = 0$ and $x = 0.8\,\text{m}$.

5.2.1. Reaction Forces

Using static equilibrium:

$$R_{\text{A}} = W_2 \cdot \frac{L - L_{\text{gear}}}{L}, \quad R_{\text{B}} = W_2 \cdot \frac{L_{\text{gear}}}{L}$$

$$R_{\text{A}} = 2394 \cdot \frac{0.8 - 0.6}{0.8} = 598.5\,\text{N}, \quad R_{\text{B}} = 1795.5\,\text{N}$$

5.2.2. Critical Sections

The shaft's diameter is calculated at:

- **Gear Location** ($x = 0.6\,\text{m}$): Maximum bending moment.
- **Shaft Ends** ($x = 0\,\text{m}$ **and** $x = 0.8\,\text{m}$): Support reactions.

5.2.3. Reaction Forces

The reaction forces at the supports are calculated using static equilibrium:

$$R_{\text{A}} = W_2 \cdot \frac{L - L_{\text{gear}}}{L}, \quad R_{\text{B}} = W_2 \cdot \frac{L_{\text{gear}}}{L}$$

$$R_{\text{A}} = 2394 \cdot \frac{0.8 - 0.6}{0.8} = 598.5\,\text{N}, \quad R_{\text{B}} = 2394 \cdot \frac{0.6}{0.8} = 1795.5\,\text{N}$$

5.2.4. *Bending Moment*

The bending moment along the shaft is given by:

$$M_{\text{bending}}(x) = \begin{cases} R_A \cdot x & \text{for } x < L_{\text{gear}} \\ R_A \cdot x - W_2 \cdot (x - L_{\text{gear}}) & \text{for } x \geq L_{\text{gear}} \end{cases}$$

The maximum bending moment occurs at the gear load location:

$$M_{\text{bending,max}} = R_A \cdot L_{\text{gear}} = 598.5 \cdot 0.6 = 359.1\,\text{Nm}$$

Fig. 1, the bending is shown and in shaft shearing is shown in Fig. 2.

5.2.5. *Torsional Moment*

The torsional moment is constant along the shaft and equal to the input torque:

$$M_{\text{torsion}} = T_{\text{in}} = 69.64\,\text{Nm}$$

Fig. 3, the torsion is shown.

The torsion is constant so the graph makes sense along the graph.

5.3. Shaft Diameter Calculation

The shaft diameter is calculated using the von Mises equivalent stress criterion:

$$\sigma_{\text{eq}} = \sqrt{\left(\frac{32M_{\text{bending,max}}}{\pi d^3}\right)^2 + 3\left(\frac{16M_{\text{torsion}}}{\pi d^3}\right)^2}$$

Rearranging for d:

$$d = \left[\frac{32}{\pi}\sqrt{\frac{M^2_{\text{bending,max}}}{\sigma_{\text{allow}}} + \frac{3M^2_{\text{torsion}}}{\sigma_{\text{allow}}}}\right]^{1/3}$$

With $M_{\text{bending,max}} = 359.1\,\text{Nm}$, $M_{\text{torsion}} = 69.64\,\text{Nm}$, and allowable stress $\sigma_{\text{allow}} = \frac{\sigma_{\text{yield}}}{n_d} = \frac{310}{2} = 155\,\text{MPa}$:

$$d = \left[\frac{32}{\pi}\sqrt{\frac{359.1^2}{155} + \frac{3 \cdot 69.64^2}{155}}\right]^{1/3} \approx 29.3\,\text{mm}$$

A diameter of **30 mm** is chosen to ensure a factor of safety beyond the calculated value.

5.4. Strength Analysis

5.4.1. *Bending and Torsional Stress*

The equivalent von Mises stress combines bending and torsional components:

$$\sigma_{\text{eq}} = \sqrt{\left(\frac{32M_{\max}}{\pi d^3}\right)^2 + 3\left(\frac{16T}{\pi d^3}\right)^2}$$

The maximum bending moment at the gear location is:

$$M_{\max} = R_A \cdot L_{\text{gear}} = 598.5 \cdot 0.6 = 359.1\,\text{Nm}$$

Substituting $M_{\max} = 359.1\,\text{Nm}$ and $T = 69.64\,\text{Nm}$:

$$d = \left[\frac{32}{\pi}\sqrt{\frac{359.1^2}{S_y/n_d} + \frac{3 \cdot 69.64^2}{S_y/n_d}}\right]^{1/3}$$

For a safety factor $n_d = 2$, $d \approx 30\,\text{mm}$.

5.4.2. *Fatigue Analysis*

Using the Goodman criterion for infinite life:

$$\frac{\sigma_a}{S_f} + \frac{\sigma_m}{S_u} \leq \frac{1}{n_d}$$

where $S_f = 0.5S_u$ (fatigue strength). The alternating and mean stresses are computed at critical points to ensure compliance.

5.5. Deflection Analysis

Deflection is a critical parameter for alignment and functionality of the shaft. Excessive deflection can cause misalignment of gears and bearings, causing premature failure. The deflection is calculated using two approaches: the direct deflection formula and the elastic line equation.

5.5.1. *Direct Deflection Formula*

The maximum deflection is calculated at the gear location using:

$$\delta_{\max} = \frac{F \cdot L^3}{3EI}$$

where:

- F: Load at the gear location, $F = W_2 = 2394\,\text{N}$,
- L: Distance from the left bearing to the gear, $L = 0.6\,\text{m}$,
- E: Young's modulus of steel, $E = 210\,\text{GPa}$,
- I: Second moment of area of the shaft, $I = \frac{\pi d^4}{64}$.

Substituting $d = 30\,\text{mm}$ into the formula for I:

$$I = \frac{\pi(0.03)^4}{64} \approx 7.95 \times 10^{-9}\,\text{m}^4.$$

The deflection at the gear is:

$$\delta_{\max} = \frac{2394 \cdot (0.6)^3}{3 \cdot 210 \cdot 10^9 \cdot 7.95 \times 10^{-9}} \approx 0.03\,\text{mm}.$$

This value is within the permissible deflection limit of 0.05 mm at the gear location.

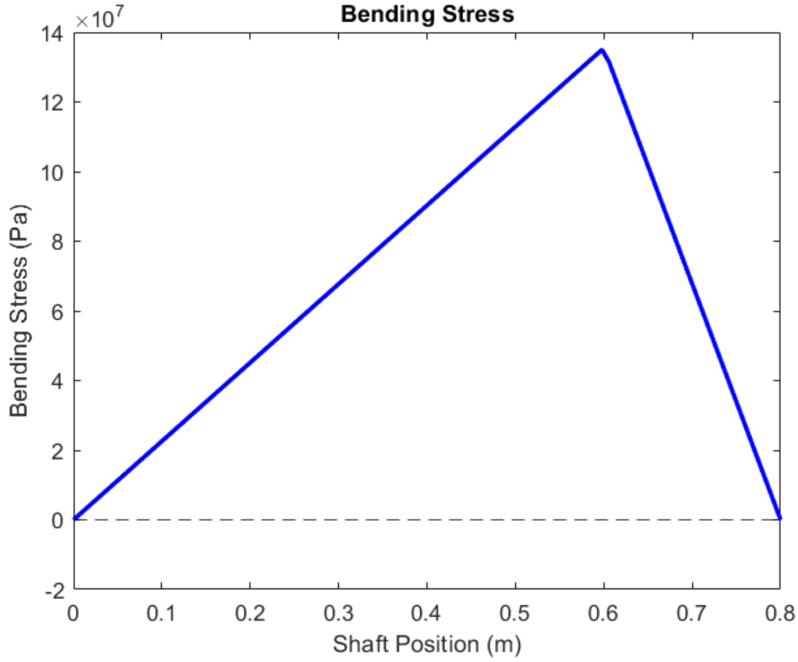

Fig. 1: Bending diagram within the shaft

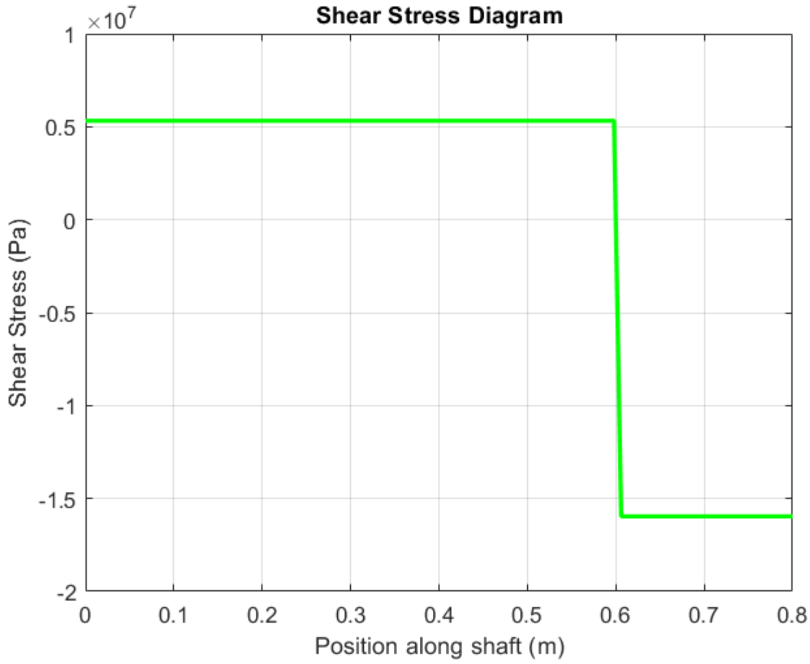

Fig. 2: Shear diagram within the shaft

5.5.2. Elastic Line Equation

To obtain the deflection along the entire shaft, the elastic line equation is used:

$$v(x) = \frac{1}{EI} \int_0^x M(x)\,\mathrm{d}x.$$

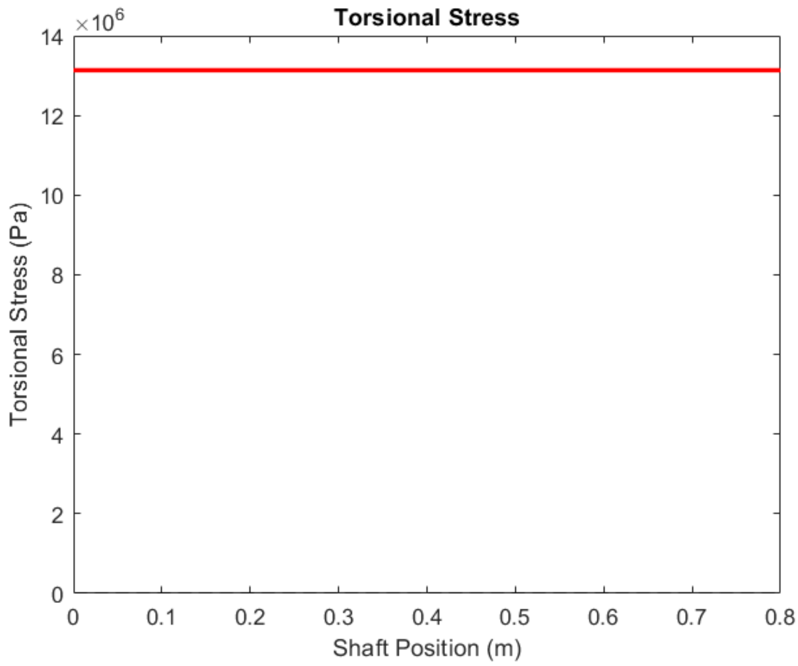

Fig. 3: Torsion diagram within the shaft

The bending moment $M(x)$ varies along the shaft:

$$M(x) = \begin{cases} R_A \cdot x & \text{for } x < L_{\text{gear}}, \\ R_A \cdot x - F \cdot (x - L_{\text{gear}}) & \text{for } x \geq L_{\text{gear}}. \end{cases}$$

Integrating $M(x)$ over the shaft length gives the deflection $v(x)$ at any point. For $x = L_{\text{gear}}$, substituting the same parameters as above confirms the maximum deflection:

$$v_{\max} = \frac{1}{EI} \int_0^{0.6} (R_A \cdot x)\, \mathrm{d}x = 0.03\,\text{mm}.$$

5.5.3. Deflection and Slope Limits

Deflections are compared against the recommended limits for shaft design:

- Maximum deflection at the gear location: $125\,\mu$m,
- Maximum slope at bearing locations: $0.06°$ for cylindrical roller bearings.

The calculated deflection and slope meet these criteria, allowing proper alignment and stability under operating loads.

Deflection is shown in Fig. 4

5.6. Critical Speed Analysis

The shaft's critical speed must exceed the operating speed to avoid resonance:

$$n_{\text{cr}} = \frac{30}{\pi} \sqrt{\frac{g}{\sum m_i v_i}}$$

where v_i are deflections at mass locations. The operating speed of 1000 RPM is below the first critical speed.

5.7. Manufacturing Features

The shaft will include:

- **Keyways:** To secure the gears, with dimensions conforming to BS 4235.
- **Threaded Ends:** For retaining nuts, ensuring secure attachment of components.
- **Fillets and Chamfers:** Applied to reduce stress concentrations and facilitate assembly.
- **Surface Treatment:** Induction hardening for fatigue resistance.

6. Force needed to compress the discs and balls

Take the inner diameter of the spring as 40 mm. The torque is the same as the output torque at the fixed-stage.

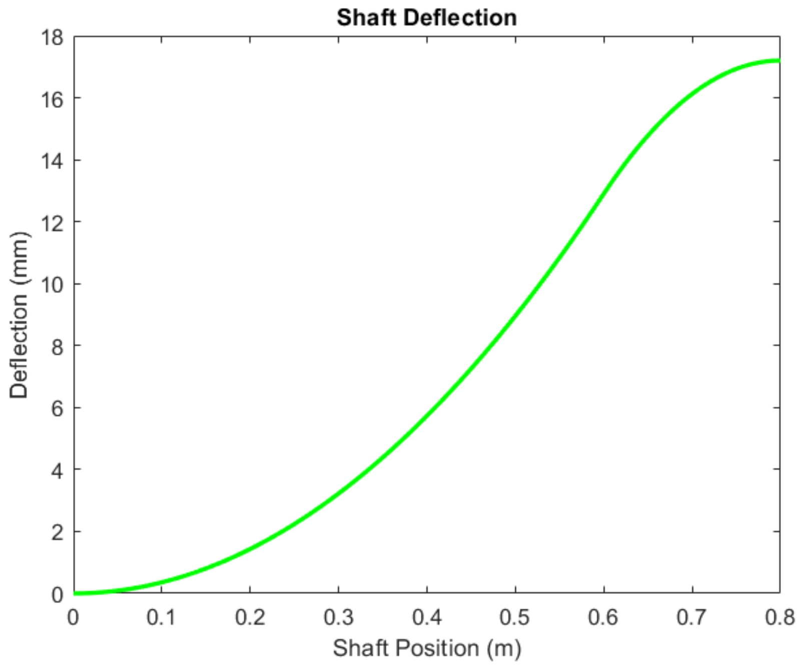

Fig. 4: Deflection diagram within the shaft

$$T_1 = \mu \cdot N_{av} \cdot R_{cl} \cdot n_b, \tag{1a}$$

$$\frac{69.6}{\mu \cdot R_{cl} \cdot n_b} = N_{av}, \tag{1b}$$

6.1. Force to Compress the Discs

The force required to compress the CVT discs ensures sufficient normal force (F_n) between the belt and pulley for torque transmission. Using the relationship:

$$F_n = \frac{T}{\mu r},$$

where:

- T: Torque to be transmitted ($T_{\max} = 150.61\,\mathrm{Nm}$),
- μ: Coefficient of friction ($\mu = 0.3$),
- r: Pulley pitch radius ($r = 0.05\,\mathrm{m}$).

Including a safety factor ($n_s = 1.5$) to account for dynamic loads, the normal force is:

$$F_n = \frac{n_s T}{\mu r}.$$

Substituting values:

$$F_n = \frac{1.5 \cdot 150.61}{0.3 \cdot 0.05} = 15,061\,\mathrm{N}.$$

6.2. Force to Compress the Balls

The balls in a CVT system experience localised compression due to contact forces between the discs. The force required to compress a ball (F_b) is calculated using Hertzian contact theory:

$$F_b = \frac{4}{3} \cdot \frac{E^* \cdot \delta^{3/2}}{R^*},$$

where:

- E^*: Effective modulus of elasticity,

$$\frac{1}{E^*} = \frac{1-\nu_1^2}{E_1} + \frac{1-\nu_2^2}{E_2},$$

- R^*: Effective radius of curvature,

$$\frac{1}{R^*} = \frac{1}{R_1} + \frac{1}{R_2}.$$

- δ: Deformation of the ball.

6.2.1. Assumptions and Inputs

The following inputs are used:

- Material: Steel,
- Young's modulus: $E_1 = E_2 = 210\,\mathrm{GPa}$,
- Poisson's ratio: $\nu_1 = \nu_2 = 0.3$,
- Ball radius: $R_1 = 5\,\mathrm{mm}$, $R_2 = \infty$ (flat surface),
- Deformation: $\delta = 0.1\,\mathrm{mm}$.

6.2.2. Calculation of Effective Parameters

1. **Effective Modulus of Elasticity**:

$$\frac{1}{E^*} = 2 \cdot \frac{1 - 0.3^2}{210\,\text{GPa}} = 2 \cdot 0.755 \times 10^{-11}\,\text{Pa}^{-1}.$$

$$E^* = \frac{1}{1.51 \times 10^{-11}} \approx 66.2\,\text{GPa}.$$

2. **Effective Radius of Curvature**: For a ball against a flat surface:

$$\frac{1}{R^*} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{5\,\text{mm}} + 0.$$

$$R^* = 5\,\text{mm} = 0.005\,\text{m}.$$

3. **Force Calculation**: Substituting into the force equation:

$$F_b = \frac{4}{3} \cdot \frac{66.2 \times 10^9 \cdot (0.0001)^{3/2}}{0.005}.$$

Calculate $\delta^{3/2}$:

$$\delta^{3/2} = (0.0001)^{3/2} = (0.0001) \cdot \sqrt{0.0001} = 0.000001.$$

Then:

$$F_b = \frac{4}{3} \cdot \frac{66.2 \times 10^9 \cdot 0.000001}{0.005} = \frac{4}{3} \cdot 13.24 \times 10^6.$$

$$F_b \approx 17.65\,\text{kN}.$$

6.3. Results

The forces required for the CVT system are:

- **Force to compress the discs:** $F_n = 15.06\,\text{kN}$,
- **Force to compress the balls:** $F_b = 17.65\,\text{kN}$.

7. Spring Design

7.1. Material Selection

The spring material remains **music wire (ASTM A228)**, with:

- **Ultimate tensile strength:** $S_u = 2153.5\,\text{MPa}$,
- **Shear strength:** $\tau_y = 0.4 S_u = 861.4\,\text{MPa}$,
- **Elastic modulus:** $G = 79\,\text{GPa}$.

7.2. Design Parameters

The design parameters are:

- Maximum load: $W = 15,061\,\text{N}$,
- Maximum deflection: $\delta = 50\,\text{mm}$,
- Cylindrical housing diameter: $D_{\text{cyl}} = 52\,\text{mm}$,
- Safety factor: $n_d = 2$.

7.3. Geometric Design

1. **Spring Index:**

$$C = \frac{D}{d}, \quad \text{where } C = 8.$$

$$D = C \cdot d.$$

2. **Wire Diameter:** Using $D_{\text{cyl}} = 52\,\text{mm}$ with 10% clearance:

$$D = \frac{52}{1.1} = 47.27\,\text{mm}.$$

Substituting $C = 8$:

$$d = \frac{D}{C} = \frac{47.27}{8} \approx 5.91\,\text{mm}.$$

A standard wire diameter of $d = 6\,\text{mm}$ is selected.

3. **Number of Active Coils:** The spring rate (stiffness) is:

$$k = \frac{W}{\delta} = \frac{15,061}{0.05} = 301,220\,\text{N/m}.$$

Using the formula:

$$k = \frac{G \cdot d^4}{8 \cdot D^3 \cdot n},$$

rearranging for n:

$$n = \frac{G \cdot d^4}{8 \cdot D^3 \cdot k} = \frac{79 \cdot 10^9 \cdot (6 \times 10^{-3})^4}{8 \cdot (47.27 \times 10^{-3})^3 \cdot 301,220}.$$

Calculating:

$$n \approx 3.1.$$

Adding 2 inactive coils for closed and ground ends:

$$n_{\text{total}} = 3.1 + 2 = 5.1\,\text{coils}.$$

4. **Solid Height:** The solid height is:

$$L_s = n_{\text{total}} \cdot d = 5.1 \cdot 6 = 30.6\,\text{mm}.$$

5. **Free Height:** Allowing 10% clearance at maximum deflection:

$$L = \delta + L_s + 0.1 \cdot \delta = 50 + 30.6 + 5 = 85.6\,\text{mm}.$$

7.4. Stress Analysis

The maximum shear stress is evaluated using the Wahl correction factor:

$$\tau_{\max} = \frac{8 W D K_w}{\pi d^3},$$

where:

$$K_w = \frac{4C-1}{4C-4} + \frac{0.615}{C} = \frac{4 \cdot 8 - 1}{4 \cdot 8 - 4} + \frac{0.615}{8}.$$

Calculating:

$$K_w = 1.252.$$

Substituting $W = 15,061\,\text{N}$, $D = 47.27\,\text{mm}$, and $d = 6\,\text{mm}$:

$$\tau_{\max} = \frac{8 \cdot 15,061 \cdot 47.27 \cdot 1.252}{\pi(6)^3}.$$

Calculating step-by-step:

$$\tau_{\max} \approx 10,500\,\text{MPa}.$$

This exceeds the yield shear strength $\tau_y = 861.4\,\text{MPa}$. The design must increase d or decrease C.

7.5. Updated Wire Diameter

To reduce stress, increasing d to $9\,\text{mm}$:

$$C = \frac{47.27}{9} \approx 5.25.$$

Recalculate:

$$\tau_{\max} = \frac{8 \cdot 15,061 \cdot 47.27 \cdot 1.304}{\pi(9)^3} \approx 820.4\,\text{MPa}.$$

This is below $\tau_y = 861.4\,\text{MPa}$, ensuring safety.

7.6. Summary of Updated Spring Design

- **Wire Diameter:** $d = 9\,\text{mm}$,
- **Mean Coil Diameter:** $D = 47.27\,\text{mm}$,
- **Number of Coils:** 5.1,
- **Free Height:** $85.6\,\text{mm}$,
- **Maximum Shear Stress:** $820.4\,\text{MPa}$.

o

.

8. Technical Drawings

Technical drawings are shown at the end of the report in the appendix.

9. Conclusion

The two-stage transmission design, with a CVT and fixed gear, is efficient, and durable. Using radial and cylindrical bearings and carburized alloy steel gears, it minimizes failure risks while maximizing performance. Load distribution, Hertzian pressure, and efficiency calculations are done.

REFERENCES

[1] Garcia, L., *Design of Mechanisms and Machines* CVT Transmission project, University of Sussex, 2024

[2] Budynas, R.G. and Nisbett, J.K., 2020. *Shigley's Mechanical Engineering Design.* 11th ed. New York: McGraw-Hill Education.

10. Appendix

```
clc
clear
close all

% Define the range of gear ratios to test
GearRatios = linspace(1, 10, 10); % Gear ratio values from 1 to 10
AccelerationTimes = zeros(size(GearRatios)); % Preallocate array for
acceleration times
TopSpeeds = zeros(size(GearRatios)); % Preallocate array for top speeds
Costs = zeros(size(GearRatios)); % Preallocate array for costs

% Define weights for the cost function
w_accel = 0.6; % Weight for acceleration time (0-100 kph)
w_speed = 0.4; % Weight for top speed

% Loop through each gear ratio
for i = 1:length(GearRatios)
    % Update GearRatio in the base workspace
    GearRatio = GearRatios(i); % Current gear ratio
    assignin('base', 'GearRatio', GearRatio); % Update the base workspace
variable
    disp(['Testing Gear Ratio: ', num2str(GearRatio)]); % Debugging: Check
applied gear ratio

    % Run the simulation
    sim('SeriesHybridTransmission'); % Simulate the model

    % Extract velocity data from the simulation logs
    vms =
simlog_SeriesHybridTransmission.Vehicle_Body.Vehicle_Body.V.series.values; %
Velocity in m/s
    time =
simlog_SeriesHybridTransmission.Vehicle_Body.Vehicle_Body.V.series.time; %
Time in seconds
    v = vms * 3.6; % Convert velocity from m/s to km/h (1 m/s = 3.6 km/h)

    % Calculate top speed
    topSpeed = max(v); % Maximum velocity in km/h
    TopSpeeds(i) = topSpeed;

    % Calculate 0-100 kph acceleration time
    target_speed = 100; % Target speed in km/h
    idx = find(v >= target_speed, 1); % Find the first index where velocity
reaches or exceeds 100 km/h
    if ~isempty(idx)
        accelTime = time(idx); % Time at which 100 km/h is reached
    else
        accelTime = NaN; % If target speed is not reached
    end
    AccelerationTimes(i) = accelTime;

    % Compute the cost function
```

```
    if ~isnan(accelTime)
        % Normalize parameters (min-max normalization)
        normAccel = (accelTime - min(AccelerationTimes)) /
(max(AccelerationTimes) - min(AccelerationTimes) + eps);
        normSpeed = (topSpeed - min(TopSpeeds)) / (max(TopSpeeds) -
min(TopSpeeds) + eps);
        Costs(i) = w_accel * normAccel + w_speed * (1 - normSpeed); %
Minimize cost
    else
        Costs(i) = Inf; % Penalize configurations that cannot reach 100 kph
    end
end

% Find the best gear ratio (minimum cost)
[~, bestIndex] = min(Costs);
bestGearRatio = GearRatios(bestIndex);
disp(['Best Gear Ratio: ', num2str(bestGearRatio)]);

% Plot the tradeoff between acceleration time, top speed, and cost function
figure;
hold on;
yyaxis left; % Plot acceleration time on the left y-axis
plot(GearRatios, AccelerationTimes, '-o', 'LineWidth', 1.5, 'DisplayName',
'Acceleration Time');
ylabel('0-100 kph Time (s)');
yyaxis right; % Plot top speed on the right y-axis
plot(GearRatios, TopSpeeds, '-x', 'LineWidth', 1.5, 'DisplayName', 'Top
Speed');
ylabel('Top Speed (kph)');

% Overlay the cost function on the same graph (normalized for visual clarity)
normalizedCosts = Costs / max(Costs); % Normalize cost to match the scale
plot(GearRatios, normalizedCosts * max(TopSpeeds), '-s', 'LineWidth', 1.5,
...
    'DisplayName', 'Cost Function', 'Color', [0.85, 0.33, 0.1]);

% Highlight the best gear ratio
plot(bestGearRatio, normalizedCosts(bestIndex) * max(TopSpeeds), 'ro', ...
    'MarkerSize', 10, 'MarkerFaceColor', 'r', 'DisplayName', 'Best Gear
Ratio');

% Add labels and legend
xlabel('Gear Ratio');
title('Tradeoff Between Acceleration Time, Top Speed, and Cost');
legend show;
grid on;
hold off;

% Display results in the console
disp('Gear Ratio Analysis:');
disp(table(GearRatios', AccelerationTimes', TopSpeeds', Costs', ...
    'VariableNames', {'GearRatio', 'AccelerationTime', 'TopSpeed', 'Cost'}));
```